Sheri D. Sheppard • Thalia Anagnos • Sarah L. Billington

# ENGINEERING MECHANICS

Modeling and Analyzing Systems in Equilibrium

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# ENGINEERING MECHANICS: STATICS MODELING AND ANALYZING SYSTEMS IN EQUILIBRIUM

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From: Sarah, Sheri, and Thalia This book is dedicated to all those who inspire us, including our partners (Ed, Jeff, and Peter), our children (Alexei, Anna, Bram, Chloe, and Portia), our teachers (past and present), and the many students we have had the privilege to teach.

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In *Engineering Mechanics: Statics*, our aim is to equip students with the knowledge, tools and good habits for solving mechanics problems in realistic contexts. Mechanics courses have historically presented engineering students with a precise, mathematical treatment of the material. This approach has appeal in that it presents mechanics as a relatively uncluttered "science." On the other hand, this material is generally of idealized cases, and students, when confronted with more realistic systems, are often at a loss as to how to proceed.

From the outset in Chapter 1 we focus on developing good problem solving habits that include being systematic about the analysis process, understanding the modeling assumptions, and developing intuition for how loads are transferred through structures and machines. This introduction of the material provides a motivational framework for the more mathematical presentation of statics found in Chapters 2–11.

Throughout this text, our emphasis is to present and illustrate:

**a.** The *physical principles* and concepts that describe non-accelerating objects. These principles and concepts are grounded in the reader's own experiences to motivate and provide a context for formal mathematical representations.

**b.** An analytical problem-solving methodology for describing and assessing physical systems, so that the reader is able to apply the principles in a systematic manner in evaluating engineered systems. Throughout the text, the methodology and its application are framed within the context of broader engineering practice.

**c.** A wide variety of problems from daily life and engineering practice. Through our "Watch-It" videos and multiple styles of artwork we demonstrate how messy-looking problems can be simplified for engineering analysis.

This online course has been written and developed explicitly with the students in mind—those in the class who are trying to get their minds around the material for the first time. Mechanics can sometimes be counterintuitive, and it can be a major frustration to those students who do not immediately relate to the logic behind the material (and this includes many of them!). Thus the presentation is a personalized one—one in which the students feel that they are having a one-on-one discussion with the authors. We do not skimp on rigor but do try to make the material accessible and, as far as we can, make it fun to learn.

# Features

The goals outlined above are supported by a number of unique features in this online course:

**Emphasis on sketching:** We emphasize the importance of communicating solutions through graphics both to enhance learning and to prepare the reader for engineering practice. Most engineering students are visual learners.<sup>1</sup> In Chapter 1 we introduce the importance of visualizing and sketching skills for the successful implementation of structured analyses, and provide guidelines for sketching objects. We further reinforce the importance of drawing through:

**a.** A full chapter (Chapter 4) devoted to the skill of drawing free-body diagrams, including drawings on engineering graph paper background that have a hand-sketched look to provide examples to students of how to document solutions. An ideal response from a reader regarding a graphical element of the text would be, "The sketch in Figure 2.3.5 made the concept more understandable AND I can create a similar drawing to illustrate the concept to someone else."

**b.** A **Draw** step included in every worked example. To reinforce the drawing concept we use "hand-drawn" figures on graph paper.

**Structured problem solving procedures:** We introduce a structured analysis procedure early in the text and use it consistently in all worked examples. These steps include explicitly listing the **Assumptions** made and the importance of the **Draw** and **Check** steps as part of a complete solution.

<sup>&</sup>lt;sup>1</sup>Felder, Richard, "Reaching the Second Tier: Learning and Teaching Styles in College Science Education." *J. College Science Teaching*, 23(5), 286–290 (1993).

Multiple paths for students to learn: Different students find they learn better in different ways and having variety is both motivating and helps deepen understanding of new concepts. We provide text to read, videos to watch, and many problems for students to tackle.

**Feedback for students and faculty:** Getting feedback is a key tool in effective learning for students and effective teaching for instructors. Online resources in *WileyPLUS* give students rapid feedback on their level of preparation, whether they understand a new concept, and on their ability to carry out more detailed calculations. At the same time, the instructor has a window into how her students are doing by getting individualized and classaverage scores to these online problems.

**Scaffolding in learning:** Statics concepts often look easy, but they can be surprisingly subtle. A strong grasp of the fundamental concepts is needed to use statics successfully to analyze systems. To develop this grasp of concepts we break them up for students into individual pieces, providing multiple opportunities to explore and master new concepts before moving on. The "Are You Ready" problems at the beginning of each chapter let students assess if they have a good understanding of the math and previously covered mechanics topics they need in order to be ready to learn the next chapter material.

Multiple study tools: To facilitate speedy access to key content, we have included review and study tools, such as Learning Objectives at the start of each chapter, and a Just the Facts section at the end of each chapter giving an overview of terms, equations, and concepts from each chapter. To the greatest extent possible, all in-text figures include *descriptive figure captions* that show at a glance what is being illustrated. *Key equations* are highlighted in yellow, and *key terms* are in bold blue type when they first appear.

# **Instructor Resources**

The following resources are available to faculty using this text in their courses:

# WileyPLUS:

The Engineering Mechanics: Statics WileyPLUS course is a new-generation online learning system

designed to address the key learning and teaching issues in today's engineering mechanics course. It includes powerful and customizable content, tools, and resources to facilitate mastery of introductory statics for students of a wide range of abilities and preparation. The system uses scaffolded practice and feedback as a means to build student competency, confidence, and commitment. The system also improves productivity and assessment of learning progress for any class size and across many sections so that instructors can focus on teaching.

Each individual element of the online experience has been crafted to become part of a larger, cohesive learning experience, one that leverages the unique capabilities available in a digital setting.

To deliver on student learning and mastery challenges, *Engineering Mechanics: Statics* implements:

- *Diagnostic assessment before each new chapter* Students are able to gauge their readiness for each new chapter—and what they may need to review further—with a brief diagnostic quiz.
- A consistent instructional cadence: tell, show, do—For each new major concept within a chapter, students will read or watch a passage that develops it, then see solved examples that apply it, and finally have an opportunity to master it through progressive, interactive exercises.
- *Scaffolded learning*—Practice exercises and a selection of homework problems use techniques such as hints, partial solutions, feedback on common mistakes, and progressive complexity to build student confidence and reinforce skills.
- Optional pathways and resources—The system facilitates differences in students' ideal learning styles. For example, they are able to choose a preferred pathway through the conceptual and example content, leveraging both video and textual content to reinforce their understanding of the material presented. All practice exercises are available to students for self-study, even if they are not formally assigned by instructors for assessment.

**Solutions Manual:** Fully worked solutions to all exercises in the text, using the same solution procedure as the worked examples.

**Electronic figures:** All figures from the text are available electronically, for use in creating your own lectures.

# Student Resources

The following resources are available to students:

**Answers to selected exercises:** The text companion site, www.wiley.com/college/sheppard, includes answers to selected exercises from the text, to help students check that they have solved the exercises correctly.

# **Commitment to Accuracy**

From the beginning we have committed to providing accurate and error-free coverage of the material. In this mission we have benefited from the help of many, many people.

While writing solutions, each solution was solved and checked at least twice, by a combination of authors, accuracy checkers, and graduate students.

All text and art were reviewed line by line by a developmental editor. A proofreader compared all corrections to final pages to confirm that any and all corrections were made. Finally, and certainly not least, the authors themselves spent countless hours checking all elements of the project at every step of the way to guarantee accuracy.

Despite our best efforts, it is possible that some errors still remain. Should anyone find anything they question, please contact the authors and we will see that any necessary corrections are made.

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In her spare time Sarah likes to read, hike, garden, and spend time with her family.

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# **CHAPTER**

# PRINCIPLES AND TOOLS FOR STATIC ANALYSIS



his text is about how to describe the forces that act on structures in equilibrium. Newton's laws of motion are used to establish mathematical relationships between the various quantities involved. These relationships enable us to predict how the quantities affect one another. After studying the material in this text, you should be able to use static analysis, which involves

- looking at a structure and seeing how it resists loads,
- 2. creating a model of the structure,
- **3.** evaluating the loads on the structure that keep it in equilibrium, and
- postulating and answering "what if" questions about the structure.

This sequence of events is illustrated in Figure 1.1.1.

Static analysis is one example of engineering analysis. More generally, engineering analysis involves performing the calculations needed to assess the behavior of a system. The basis for these calculations is often physical principles from chemistry and physics. This chapter presents background material for static analysis.

### L . E Α R N П Ν G 0 В E С V E S J т Π

On completion of this chapter, you will be able to:

- Summarize the steps of the product realization process and an engineering analysis procedure. (1,1)
- State Newton's three laws of motion. (1.2)
- Convert between SI and USCS units. (1.3)
- Represent vectors. (1.4)
- Recognize the different types of drawings used in engineering analysis and basic guidelines for creating them. (1.5)
- Describe good problem-solving habits. (1.6)
- State the overall goal of this text. (1.7)



**Figure 1.1.1** Engineer using analysis to answer a question.

# 1.1 HOW DOES ENGINEERING ANALYSIS FIT INTO ENGINEERING PRACTICE?

# **Learning Objective:** Summarize the steps of the product realization process and an engineering analysis procedure.

There are some 1.5 million practicing engineers in the United States; this is less than 1% of the U.S. population. Engineers create the products and systems that we interact with daily. They create products that improve our quality of life (surgical devices, air-scrubbers in smoke stacks), entertain us (roller blades, roller coasters, electric trains, bikes), and educate us (LCD projection systems, computers). Engineers also create the systems that extend our reach from our planet's surface to the bottom of the ocean and to distant planets.

The process by which engineers design and manufacture these products and systems is referred to as the **product realization process** and may extend over months (less than six months for disk drives), years (for automobiles or bridges), or even decades (as in the case of the space station).

Any product or system begins with someone identifying an initial *client need* (the design problem). This need may arise from the market, the development of new technology, the demand for more sophisticated engineered systems or simply the President of the United States stating, "We will go to the moon before the end of the decade." Engineers design a product or system to solve some problem. Identification of a problem includes development of a list of design requirements. These design requirements are benchmarks used to evaluate progress toward a design solution, as well as the effectiveness of the final design solution. They may have to do with, for example, the final design's performance, appearance, time-to-market, cost, ease of manufacture, safety, impact on the environment, or ability to meet national or international standards.

Listing of design requirements is followed by generation of ideas on how to address the need or problem. These early ideas are referred to as *design concepts*, and this phase of the product realization process is known as *conceptual design*.

Conceptual design is followed by *preliminary design*, where some of the concepts are developed further and some are discarded. Often the decision to continue with or discard a concept is based on an evaluation of how well the concept meets the design requirements. Evaluation may involve calculations and/or building prototypes (physical or virtual) of the concept. Typically, preliminary design ends with the selection of a single concept that will be detailed and refined in the next phase of design (called detail design).

Decisions made during *detail design* about specific configurations of components, types of materials, size of connections, methods of manufacturing, and so on, are often based on analysis to confirm that design decisions and choices continue to meet the design requirements. The analysis may involve numerical modeling and simulation. Building and testing of prototypes may also be involved.

Detail design results in a *comprehensive description* of the product or system. This description consists of drawings, complete fabrication specifications, and supporting documentation that describes the design decisions. It should also include analysis details and test results that support these decisions.

Detail design is followed by production, in which the product or system is constructed or manufactured. Here engineers oversee the process to verify that the final product meets the design requirements. Analysis may be used in this verification.

The product realization process that we have described may sound like a linear, sequential process, with one phase connecting to the start of another phase. In reality the process is a continuous loop, as suggested in **Figure 1.1.2**. For example, new design requirements may be generated later in the process as additional details of the design are being worked out. Also the real problem being solved may not be identified until well into the conceptual phase of design, or two competing concepts may be carried into detail design before a decision is made as to which one will be produced.

Regardless of where in the product realization process flowchart an engineer is working, he or she is likely to be involved in verifying and justifying decisions about the product. Engineering analysis is one of the main tools the engineer will use. The major steps in engineering analysis are summarized as an engineering analysis procedure (see Box 1.1).

In carrying out engineering analysis it is critical to simultaneously consider the physical situation and the mathematical model of the physical situation. The mathematical model allows us to understand and predict performance of the physical situation. At the same time, any model is only an approximation of the physical situation, and so is an estimate of real performance. One of the challenges in undertaking engineering analysis is learning to appropriately model a physical situation to obtain insights into its approximate performance.



**Figure 1.1.2** Product realization process flowchart.





# **1.2 PHYSICS PRINCIPLES: NEWTON'S** LAWS REVIEWED

# Learning Objective: State Newton's three laws of motion.

The physical principles that underlie engineering analysis in this text are Newton's three laws of motion:<sup>1</sup>

- First Law: An object will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion) if the resultant force acting on the object is zero. Another way of stating the same law is that an object originally at rest, or moving in a straight line with constant velocity, will remain in this state provided the object is acted on by balanced forces.
- Second Law: If the resultant force acting on an object is not zero, the object will have an acceleration proportional to the magnitude of the resultant force and in the direction of this resultant force.

<sup>1</sup>The man most immediately responsible for what you'll be learning in this book is Sir Isaac Newton. Even among geniuses, Newton stands out. He needed a new mathematical approach to handle his investigations and so he invented calculus. That same year, he revolutionized optics by realizing that white light is made up of a spectrum of colors. And, to top it all off, he laid down his three laws of motion. Even more amazing, he did all of this when he was in his early twenties while taking a short break from London in order to avoid the plague.

*Third Law:* The forces exerted by two objects on each other are equal in magnitude and opposite in direction.

In this text we use the first and third laws extensively to describe situations where objects are at rest or are moving at constant velocity as a result of being acted on by balanced forces. We call these situations "static." This text is about static analysis, which is often referred to simply as **statics**. Statics can be used to design and describe a wide array of engineered systems, from the propulsion of bicycles (as described in Appendix D) to the tension in the cables in a suspension bridge (as described in Appendix E).

Closely related to statics is **dynamics**, the area of engineering that also embodies analysis based on Newton's laws except that the object is moving at a nonconstant velocity, an acceleration, as described by Newton's second law. In mathematical terms, the second law says that if an object is acted upon by an unbalanced force F, the object experiences acceleration a in the same direction as the force. The acceleration is proportional to the force (and the proportionality factor is the mass m of the object):

$$\boldsymbol{F} = m\boldsymbol{a} \tag{1.1}$$

The bold italic notations F and a denote that these are vector quantities. Dynamics is usually covered in a separate course apart from statics.

Together statics and dynamics make up the study of "rigid body mechanics." A **rigid body** is a combination of a large number of particles in which all the particles remain at a fixed distance from one another before, during, and after a force is applied to the object. As a result, the material properties of any object that is assumed to be rigid will not be considered when analyzing the forces acting on the object. In most cases, the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis or preliminary design. Detail design requires full investigation of the deformations.

# **1.3** PROPERTIES AND UNITS IN ENGINEERING ANALYSIS

# Learning Objective: Convert between SI and USCS units.

Static analysis involves quantifying, manipulating, and measuring properties of objects. The properties we are concerned with are length, time, mass, and force:

*Length* is a description of distance.

- *Time* is conceived as a succession of events. Although the principles of statics are time-independent, this quantity does play an important role in the study of dynamics.
- *Mass* is a property of matter by which the action of one object can be compared with the action of another. This property manifests itself as a gravitational attraction between two bodies and provides a quantitative measure of the resistance of matter to a change in velocity.

Force is considered as a push or pull exerted by one object on another.

Name	Standard Unit of Length	Standard Unit of Time	Standard Unit of Mass	Standard Unit of Force
International System of Units (SI)	meter (m)	second (s)	kilogram (kg)	newton (N)*
U.S. Customary System of Units (USCS)	foot (ft)	second (s)	slug**	pound (lb)

Table 1.1	Standard	Measures
-----------	----------	----------

\*derived quantity, based on meter, second, and kilogram, as discussed below  $(N = \frac{kg \cdot m}{s^2})$ 

\*\*derived quantity, based on foot, second, and pound, as discussed below (slug =  $\frac{1b \cdot s^2}{6}$ )

In working with these quantities we need consistent and standard measures—these are provided by the **International System of Units** (abbreviated SI after the French Le Système International d'Unités) and the **U.S. Customary System of Units** (USCS), as summarized in **Table 1.1**. The SI system is the accepted national standard of measurement in all countries except Myanmar, Liberia, and the United States.

# **SI Units**

As shown in **Table 1.1**, the standard measure of length in the SI system is the **meter**, which is roughly the length from an adult's nose to his or her extended finger tips. Often engineers deal with lengths that are much larger (e.g., Earth's radius) or smaller (e.g., the thickness of a sheet of paper) than a meter; therefore, it may be more appropriate to deal with multiples or submultiples of the meter. We denote these multiples or submultiples with the prefixes listed in **Table 1.2**. For example, the

# Table 1.2 SI Prefixes\*

Factor	Prefix	Symbol
$10^{18}$	exa-	Е
$10^{15}$	peta-	Р
$1\ 000\ 000\ 000\ 000 = 10^{12}$	tera-	Т
$1\ 000\ 000\ 000 = 10^9$	giga-	G
$1\ 000\ 000 = 10^6$	mega-	М
$1\ 000 = 10^3$	kilo-	k
$100 = 10^2$	hecto-	h
$10 = 10^1$	deka-	da
$0.1 = 10^{-1}$	deci-	d
$0.01 = 10^{-2}$	centi-	с
$0.001 = 10^{-3}$	milli-	m
$0.000\ 001 = 10^{-6}$	micro-	μ
$0.000\ 000\ 001 = 10^{-9}$	nano-	n
$0.000\ 000\ 000\ 001 = 10^{-12}$	pico-	р
$0.000\ 000\ 000\ 000\ 001 = 10^{-15}$	femto-	f
$0.000\ 000\ 000\ 000\ 000\ 001 = 10^{-18}$	atto-	а

\*Prefixes commonly used in this text are shown in boldface type.

mean radius of Earth is  $6.37 \times 10^6$  m or 6370 km, and a sheet of paper is  $1 \times 10^{-4}$  m or 0.1 mm thick.

The standard measure of mass in the SI system is the **kilogram** (kg), defined as the mass of a particular platinum-iridium cylinder kept at the International Bureau of Weights and Measures near Paris. From **Table 1.2**, we see that the prefix of "kilo" means that this standard has a mass of 1000 grams. Engineers work with a range of mass sizes, from the very large (mass of a Boeing 787) to the very small (mass of a white blood cell). The standard measure of time is the **second** (s).

The standard unit of force in the SI system is the **newton** (N). One newton is equal to the force required to give 1 kilogram of mass an acceleration of  $1 \text{ m/s}^2$ . We will have a lot more to say about forces in Chapter 2.

In the SI system, length, mass, and time are the fundamental properties, and force is a derived quantity from Newton's second law. By Newton's second law (1.1), one newton (1 N) of force equals  $[1 \text{ kg}][1 \frac{\text{m}}{\text{s}^2}] = [\frac{\text{kg} \cdot \text{m}}{\text{s}^2}]$ . Guidelines for working with SI prefixes and units are given in **Box 1.2**.

# **U.S. Customary Units**

The standard measure of length in this system is the **foot**, as shown in **Table 1.1**. The standard measures for time and force are the **second** and **pound**, respectively.

In the U.S. Customary system, the fundamental properties are length, force, and time. The standard unit of mass in the U.S. Customary system is called the **slug** and is derived from the foot, second, and pound using Newton's second law. One slug is equal to the amount of matter that is accelerated at 1 ft/s<sup>2</sup> when acted upon by a force of 1 pound (1 slug =  $1 \text{ lb} \cdot \text{s}^2/\text{ft}$ ).

No matter which system of units you are working with, it is imperative that you *use consistent units*. For example, if you are using kilometers as the measure of length, make sure that you use kilometers consistently for all measures of length in the problem. Do not mix with feet or miles. Sometimes you may need to convert quantities from one measurement system to another; **Table 1.3** lists some conversion factors for going between U.S. Customary units and SI units.

Converting from U.S. Customary to SI					
Quantity	U.S. Customary	To SI multiply by			
Force	lb	4.4482 N/lb			
Mass	slug	14.5938 kg/slug			
Length	ft	0.3048 m/ft			
Converting from SI to U.S. Customary					
Quantity	SI	To U.S. Customary multiply by			
Force	Ν	0.2248 lb/N			
Mass	kg	0.06852 slug/kg			
Length	m	3.2808 ft/m			

	Table 1.3	Conversion	Factors
--	-----------	------------	---------

# Box 1.2: Guidelines for Working with SI Prefixes and Units

- 1. Unit symbols are always written in lowercase letters, with the following exceptions: symbols for some prefixes and symbols named after an individual are capitalized (e.g., N for newton).
- 2. Unit symbols are never written with a plural "s" because this may be confused with the unit for second (s).
- 3. Compound prefixes should not be used. For example, k  $\mu$ m (kilo-micrometer) should be expressed as mm (millimeter) since  $1(10^3)(10^{-6})$  m =  $1(10^{-3})$  m = 1 mm.
- 4. The exponential power given for a unit having a prefix refers to both the unit and its prefix (e.g.,  $mm^2 = (mm)^2 = mm \cdot mm$ ).
- 5. In engineering notation, exponents are generally displayed in multiples of three. This convention facilitates conversion to the appropriate prefix. For example, 4.0(10<sup>3</sup>) N can be rewritten as 4.0 kN.
- 6. Quantities defined by several units that are multiples of one another are separated by a dot to

avoid confusion with prefix notion (e.g.,  $N = kg \cdot m/s^2 = kg \cdot m \cdot s^{-2}$ ). The dot notation differentiates m·s (meter-second) from ms (millisecond).

- 7. Avoid prefixes in the denominator of composite units. For example, write kN/m rather than N/mm. The exception to this rule is the kilogram (kg); since it is the base unit of mass, it is fine to use it in the denominator (e.g., write Mm/kg rather than km/g).
- 8. When calculating, convert all prefixes to powers of 10. For example,  $(100 \text{ kN})(200 \ \mu\text{m}) = [100(10^3) \text{ N})][200(10^{-6})\text{m}] = 20,000(10^{-3}) \text{ N} \cdot \text{m}$ . Then express the final result using a single prefix combined with a numerical value between 0.1 and 1000: 20,000(10^{-3}) \text{ N} \cdot \text{m} becomes 20 N  $\cdot$  m.
- Minutes, hours, days, and so forth are used for multiples of the second. Plane angular measurement is made using radians (rad) or degrees (°).

# EXERCISES 1.3

**1.3.1.** [\*] Derive conversion factors for changing the following U.S. Customary units to their SI equivalents:

- a. Pressure, lb/in.<sup>2</sup>
- **b.** Force, kip
- **c**. Volume,  $ft^3$
- **d.** Area, in.<sup>2</sup>

**1.3.2.** [\*] Derive conversion factors for changing the following SI units to their U.S. Customary equivalents:

- **a**. Pressure,  $N/m^2$  (pascal)
- b. Pressure, MPa (Megapascal)
- **c**. Volume, m<sup>3</sup>
- **d**. Area,  $mm^2$

**1.3.3.** [\*] Jamaican sprinter Asafa Powell set the world record for the 100-meter dash on May 27, 2010. His time was 9.07 seconds. Calculate his average speed in m/s, ft/s, and mph.

**1.3.4.** [\*] Calculate the percent difference between the mile and the metric mile (1500 meters).

**1.3.5.** [\*] The world best performance in the women's marathon is 2:17:42, set by Paula Radcliffe of the United Kingdom on April 17, 2005 in the London Marathon. On

average, how long did it take her to run each mile? What was her average speed in m/s? A previous best performance was 2:18:47, turned in by Catherine Ndereba from Kenya. (The race was run in Chicago on October 7, 2001.) How much faster did Paula Radcliffe run each mile of the race?

**1.3.6. [\*]** In the heavyweight division, Russian Aleksey Lovchev holds the world record for the clean and jerk. He lifted a mass of 264 kg. Calculate the mass in slugs. What is the corresponding weight in newtons and pounds? How many people would it take to clean and jerk a Porsche 911 if they were all as strong as Aleksey Lovchev? (Make sure to document your source for weight data.)

**1.3.7.** [\*] When a certain linear spring has a length of 180 mm, the tension in it is 170 N. For a length of 160 mm, the compressive force in the spring is 120 N.

**a.** What is the stiffness of the spring in SI units? In U.S. Customary units?

**b.** What is its unstretched length in SI units? In U.S. Customary units?

**1.3.8.** [\*] Complete the following two tables:

Event	Meters	Centimeters	Inches	Feet	Miles
High jump	2.45		96.46		
Pole vault	6.16			20.21	
Long jump			352.36		
Triple jump			720.08	60.01	1.14E-02
Shot put	23.12			75.85	
Discus throw		7408			
Hammer throw		8674	3414.96	284.58	
Javelin throw	98.48			323.10	

MEN'S World Records for Selected Field Events

WOMEN'S World Records for Selected Field Events

Event	Meters	Centimeters	Inches	Feet	Miles
High jump	2.09			6.86	
Pole vault		506	199.2		
Long jump				24.67	4.67E-03
Triple jump	15.50		610.2		9.63E-03
Shot put		2263		74.25	
Discus throw			3023.6	251.97	
Hammer throw			3192.1	266.01	
Javelin throw				237.14	

# **1.4** COORDINATE SYSTEMS AND VECTORS

# Learning Objective: Represent vectors.

# **Coordinate Systems**

In working with physical objects it is useful to specify information about them relative to a **Cartesian coordinate system**, which uses three axes that are orthogonal to one another, as shown in **Figure 1.4.1***a*. In addition, the system is **right-handed**. In a right-handed system, if you point the fingers of your right hand in the direction of the positive *x* axis and bend them (as in preparing to make a fist) toward the positive *y* axis, your thumb will point in the direction of the positive *z* axis, as shown in **Figure 1.4.1***b*.

The assignment of coordinate axes is often a matter of convenience, and the choice is frequently up to the engineer. The logical choice is usually indicated by the geometry of the situation. For example, when the principal dimensions of a system or structure are given in the horizontal and vertical directions, the assignment of coordinate axes in these directions is generally convenient (**Figure 1.4.2***a*). If the structure and/or the forces are not aligned with the horizontal and vertical directions, alternative orientations of the coordinate axes may be appropriate, as shown in **Figure 1.4.2***b*.

# **Scalar and Vector Quantities**

Static analysis deals with two kinds of quantities—scalars and vectors. Scalar quantities can be completely described with a magnitude (number



**Figure 1.4.1** *xyz* coordinates arranged in right-handed manner.





Figure 1.4.3 A position vector.



**Figure 1.4.4** (*a*) Two vectors to be added; (*b*) vector addition using the parallelogram law.

Figure 1.4.2 Various orientations of coordinate axes.

only) and associated units. Examples of scalar quantities are mass, density, length, area, volume, speed, energy, time, and temperature. In mathematical operations, scalars follow the rules of elementary algebra. Scalars in this text are represented with italic type (V).

In contrast to scalars, **vector quantities** have both magnitude (with units) and direction, and obey the parallelogram law of addition, as described below. Examples of vector quantities are velocity, acceleration, momentum, force, moment, and position.

A vector is typically represented in drawings by an arrow with a head and a tail (**Figure 1.4.3**). The direction from the tail to the head of the arrow represents the direction of the vector, and the length of the arrow is often drawn proportional to the magnitude of the vector. The magnitude of the vector is generally written next to the arrow.

In this text, vector quantities are distinguished from scalar quantities through the use of boldface italic type (V). In longhand writing, a vector may be denoted by drawing a "half arrow" above the letter,  $\overline{V}$ . Euclidean norm bars surrounding the vector symbol are used to denote the *magnitude* of a vector. Thus, the magnitude of the vector Vis denoted by ||V||, or  $||\overline{V}||$  (in longhand).

As mentioned above, vectors obey the **parallelogram law of addition**. This means that the two vectors  $V_1$  and  $V_2$  in Figure 1.4.4*a* can be replaced by an equivalent vector V that is the diagonal of a parallelogram